Machine Dynamics and Chaos in Grinding with Rattling

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Dynamical analysis of the snagging operation is introduced. The theory on the cutting is used as described by Yokouchi and the dynamics of the machine is determined as quasi-bilinear impact system with hysteresis and treated with numerical methods. The wheel rattling on the work is identified as either chaotic or quasi-periodic in a wide range of operating conditions. Hence, the arguable reasons for the empirically known facts of the uniform wear distribution around the wheel and the unpredictable qualitative wheel wear characteristics were identified.

1. Introduction

Several decades ago, in several steel industries in Japan, a set of wear data obtained for planned same grinding conditions on snagging operation have been showed contradictory results on wheel wear behavior and no conclusions were reached. Therefore, in this study, we made an attempt to describe some arguable reasons with the help of non-linear dynamics.

2. Simple impact dynamical model.

Modeling and treating the exact snagging operation is extremely difficult because of the presence of impacting non-linearity, delayed forcing terms (lobes interacting with the work) and time dependent coefficients[1)] (cutting stiffness) and so on. The other disturbances such as the belt slipping, the belt vibration and the effect of rattling joints were neglected.

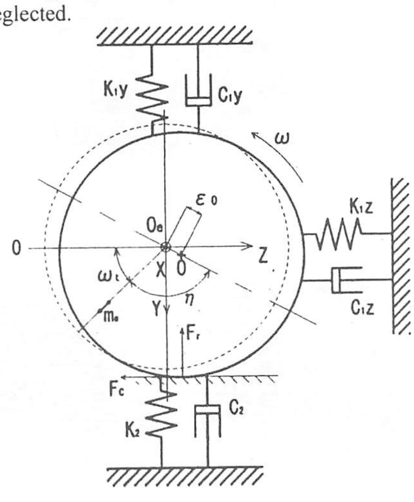


Fig.1. Simple schematic of the impact model for the snagging operation

The equation of motion can be defined simply as two portions referred as jumping and cutting with various non-dependent natural frequency terms. Referring to **Fig. 1**,

In the case of Jumping

$$\ddot{\phi}_Z + 2\xi_{zA}\omega_{nzA}\cdot\dot{\phi}z + \omega_{nzA}^2\cdot\phi_Z + \alpha_z\cdot\dot{\phi}_Y = F_y\sin(\omega t) \qquad (1\text{-}1)$$

$$\ddot{\phi}_Y + 2\xi_y\omega_{ny}\cdot\dot{\phi}_Y + \omega_{ny}^2\cdot\phi y - \alpha_y\cdot\dot{\phi}_Z = -F_z\cos(\omega t) \qquad (1\text{-}2)$$

In the case of Cutting

$$\ddot{\phi}_Z + 2\xi_{zC}\omega_{nzC}\cdot\dot{\phi}_Z + \omega_{nzC}^2\cdot\phi_Z + \alpha_z\cdot\dot{\phi}_Y = F_y\sin(\omega t) - T_y\sin(\omega t + \eta) \qquad (2\text{-}1)$$

$$\ddot{\phi}_Y + 2\xi_y\omega_{ny}\cdot\dot{\phi}_Y + \omega_{ny}^2\cdot\phi_Y - \alpha_y\cdot\dot{\phi}_Z = -F_z\cos(\omega t) - \frac{1}{2}\{T_z\sin(\omega t + \eta) + P\cdot\phi_Z\} \qquad (2\text{-}2)$$

Where

$F_z = \dfrac{lme\omega^2}{2I_{dy}}$, $\alpha_y = \dfrac{\omega I_p}{2I_{dy}}$, $\omega_{ny} = \sqrt{\dfrac{G_y}{2I_{dy}}}$, $\xi_y = \dfrac{C_y}{2}\sqrt{\dfrac{1}{2G_yI_{dy}}}$, $T_z = \dfrac{lK_2\varepsilon_0}{2I_{dy}}$, $P = \dfrac{l^2K_2}{2I_{dy}}$

$F_y = \dfrac{lme\omega^2}{2I_{dz}}$, $\alpha_z = \dfrac{\omega I_p}{2I_{dz}}$, $\omega_{nzA} = \sqrt{\dfrac{G_z}{2I_{dz}}}$, $\xi_{zA} = \dfrac{C_z}{2}\sqrt{\dfrac{1}{2G_zI_{dz}}}$, $\omega_{nzC} = \sqrt{\dfrac{G_z'}{2I_{dz}}}$

$\xi_{zC} = \dfrac{C_z}{2}\sqrt{\dfrac{1}{2G_z'I_{dz}}}$, $T_y = \dfrac{lK_2\varepsilon_0}{2I_{dz}}$, $C_y = C_{1z}l^2$, $G_y = K_{1z}\cdot l^2$, $C_z = C_{1y}l^2$, $G_z = K_{1y}l^2$

$G_z' = (K_{1y} + K_2)l^2$

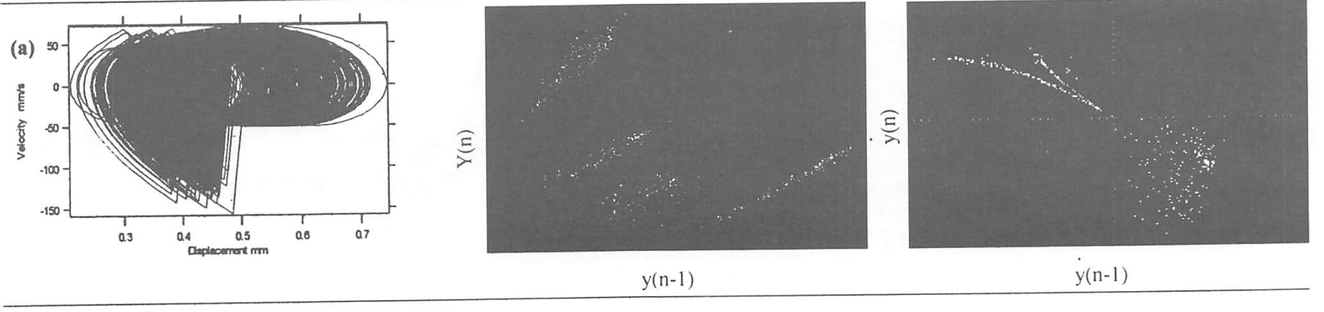
This system co-operates with gyroscopic effects due to the high inertia and the less stiffness of the machine with comparatively high speed, unfamiliar to most of the other stock removing operations such as milling and fine grinding. The cutting and the machine parameters are tabulated in **Table 1**.

Table1. Machine and grinding parameters

Wheel	A20MB
Work	.SCM4(JIS)
Wheel speed (ω)	3140 r.p.m
Feed rate (v)	31.7mm/s
Work diameter	172mm
Wheel edge radius (r)	86mm
Machine damping (C_y, C_z)	0.532Ns/mm,2.02Ns/mm
Machine stiffness (K_y, K_z)	1.38kN/mm,19.6kN/mm
Load applied on the wheel (W)	470 N
I_{dy}	164.5 kgmm^2
I_{dz}	133.5 kgmm^2
I_p	1.65kgmm
l	110 mm

3. The chaotic behavior of the system

The return maps of **Fig.2(a)** shows fractal like structure and the rest indicates more concentrated patterns. Therefore, it can be identified that the quasi-periodic behavior of the dynamical system changes to strange attractor that could be chaotic, with the changes of the machine stiffness.



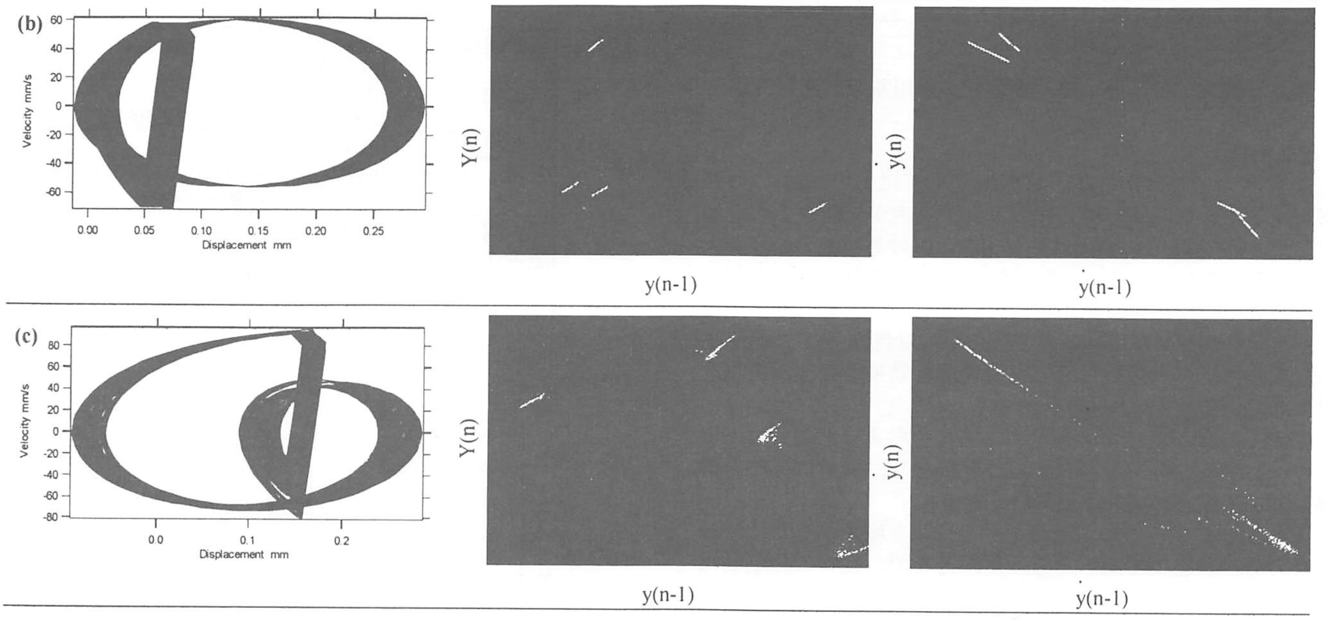


Fig.2 First and second return maps obtained for the vibration of wheel center for different stiffness ratio $\gamma = K_2 / K_1$ conditions. for **(a)** γ **=72 ,(b)** γ **=50,(c)** γ **=12.5.** K_2 is kept constant at 100kN/mm.

The predicted values of the largest Lyapunov exponent and the correlation fractal dimensions are listed in **Table 2**.The respective embedding dimension is determined as 5 which implies that the vibration can be categorized as system of having low dimensional chaos. Tabulated values include the calculation error terms as well. The experimental results show a higher Lyapunov exponent due to the noise in the vibration signals.

Table2. Estimated results

	Experiment	Simulation
Largest Lyapunov exponent	0.269∓0.032	0.059∓0.007
Correlation fractal dimension	2.80∓0.372	1.892∓0.059

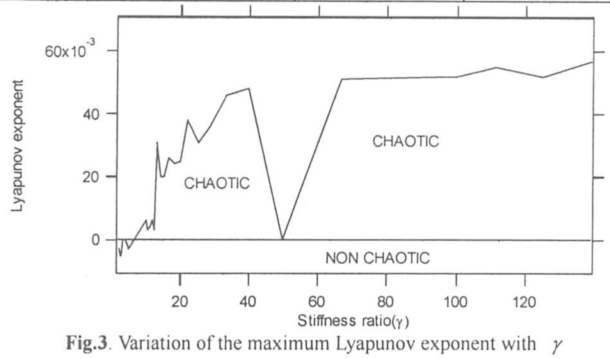


Fig.3. Variation of the maximum Lyapunov exponent with γ

According to the results shown in **Fig.3**, the tendency for chaotic dynamics to occur is higher when the machine stiffness is lower. There are no significant variations of the Lyapunov exponent, which is an indication of the insignificancy of the damping on the chaotic behavior of the system for lower damping region **(Fig.4).**

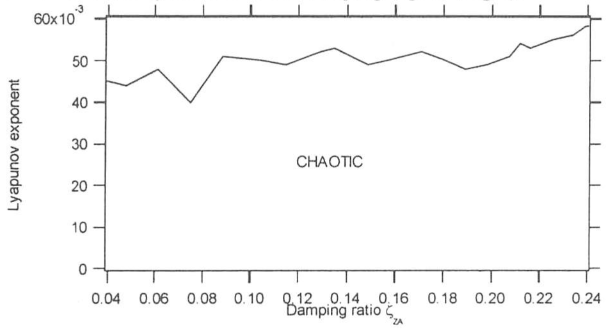


Fig.4. Variation of the maximum Lyapunov exponent with damping factor

4.The wear distribution behavior

Fig.5 and **Fig.6** show the wheel engaging position with the work. Both emphasize that the engaging position changes continuously while the grinding progress. Therefore, the uniform wheel wear behavior is identified of the system seemingly linear.

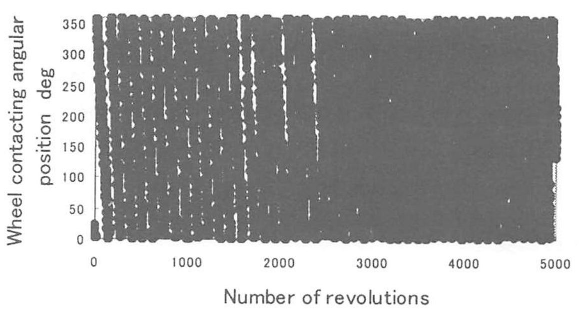


Fig.5. The distribution of the wheel engaging angular position up to 5000 revolutions for auto balancer

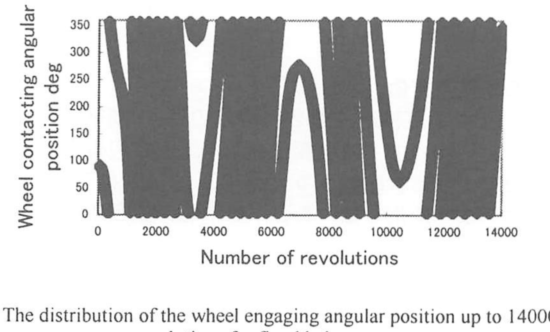


Fig .6 The distribution of the wheel engaging angular position up to 14000 revolutions for fixed balancer

5.Conclusions

Grinding with rattling is modeled and the dynamical behavior of the machine is investigated by numerical methods. The solution has shown of having strange attractors and identified as chaotic for wide range of operating conditions. Hence, the reasons for the distributed wear behavior of the wheel and the unpredictability of the wear pattern were identified.

References

1) H.Yokouchi, Y.Onouchi, K.Kikuchi: Study on the Snagging-Theoretical Analysis of Grinding Mechanisms When Wheel Rattles on the Work, JSPE, Vol 47-7, July (1983)90 (In Japanese)