Approximation and Compensation of Delay in Analog Control Systems

○Andri Mirzal, Shinichiro Yoshii and Masashi Furukawa Graduate School of IST Hokkaido University

Time delays are components that make time-lag in systems response. They arise in physical, chemical, biological and economic systems, as well as in the process of measurement and computation. In this study, we examine and compare error rates of seven polynomial series which approximate the delays. Moreover we study PID controller as delay compensation scheme, and measure its characteristics under compensated delay with using two tuning methods, "iterative method" and Ziegler-Nichols method.

1. Introduction

Delay in control systems can be defined as time-interval between an event start in one point and its output in another point within the systems [1]. Time delays always reduce minimum phase systems stability. Delays can be caused by transportation and communication lag, sensors response delay in control systems, time to generate control signals, and system parameters approximation with FOLPD (First Order Lag plus Time Delay) [2].

In analyzing control systems with time delays, we frequently have to use delay approximation with polynomial series. Time delays approximation with polynomial series make it possible to use analysis methods that is common in analysis of non time delays control systems. In some analysis methods like Routh-Hurwitz stability criterion and root locus analysis, approximation can't be avoided.

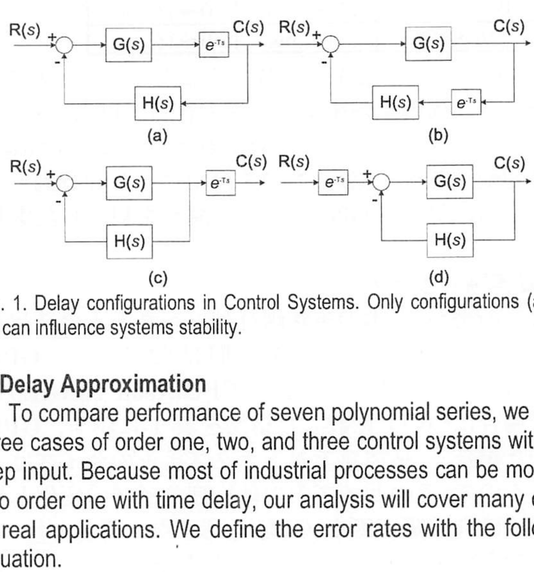

Fig. 1. Delay configurations in Control Systems. Only configurations (a) and (b) can influence systems stability.

2. Delay Approximation

To compare performance of seven polynomial series, we study three cases of order one, two, and three control systems with unit step input. Because most of industrial processes can be modeled into order one with time delay, our analysis will cover many cases in real applications. We define the error rates with the following equation.

$$E = \int_0^\infty \left| D(s) - \overline{D(s)} \right| ds \quad (1)$$

where $D(s)$ is delay in domain frequency, and $\overline{D(s)}$ is delay approximation with series.

Results are shown in Table 1. This table shows that DFR series gives the minimum errors for order one and order three. In case of order two, Product series gives the minimum errors and DFR is the second. However, the difference between these two series is very small. Furthermore, DFR series has the minimum averaged errors in three cases. So, it is concluded in general DFR series has the best performance among seven analyzed series.

If we investigate the error rates plots, we can find the interesting patterns, especially in cases of order two and three, in which there is a breaking point. If delay is smaller than the delay at the breaking point, all seven series give relatively the same error rates. And if it is bigger than this value, the error rates will diverge. Unfortunately, in case of order one, such a breaking point does not really exist.

In case of order one, Marshall series diverges immediately after leaving the first point at delay 0.01 second, following by Taylor series at 0.3 second. But at delay 3 second we can see such a breaking point at which the series diverges. In the case of order two the breaking point is at delay 0.3 second and in order three is at delay 1 second.

Also, it is pointed out that, in all three cases, the series give relatively the same error rates except for Marshall and Taylor series. So, it is advisable not to use these two series for delay approximations.

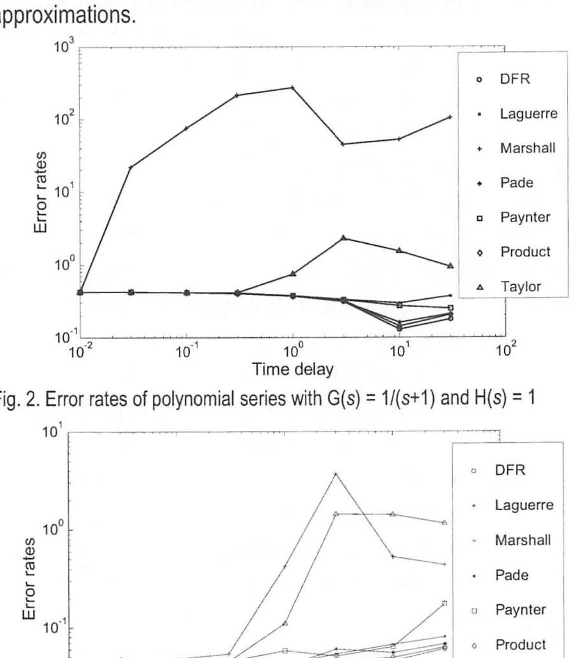

Fig. 2. Error rates of polynomial series with $G(s) = 1/(s+1)$ and $H(s) = 1$

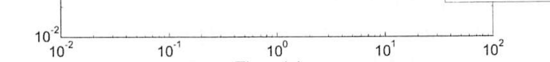

Fig. 3. Error rates of polynomial series with $G(s) = 1/(s^2+1.4s+4)$ and $H(s)=1$

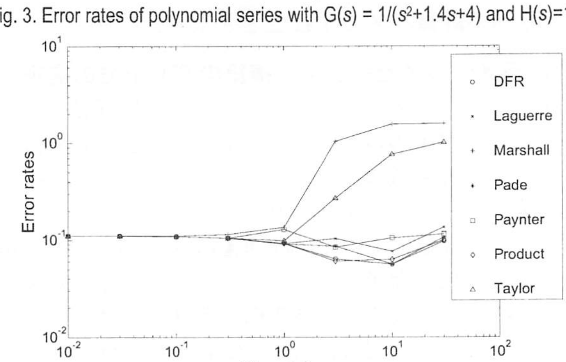

Fig. 4. Error rates of polynomial series with $G(s)=1/(s^3+3s^2+4s+2)$, $H(s)=1$

Table 1. Average errors of seven polynomial series for all cases

Series	Order One	Order Two	Order Three	Average
Taylor	0.897775	0.535338	0.322543	0.585219
Pade	0.34035	0.050875	0.096875	0.1627
Marshall	98.06926	0.660313	0.59705	33.10887
Product	0.338063	**0.047138**	0.093781	0.159661
Laguerre	0.38	0.053425	0.105359	0.179595
Paynter	0.362775	0.066375	0.108758	0.179303
DFR	**0.331525**	0.04759	**0.093026**	**0.15738**

3. PID Controllers

PID controllers are main controllers in industries (more than 90%[3]) that can compensate most of delays in industrial processes.

It has high level of robustness, and is easy to operate and understand because of its structure simplicity. On the other hand, PID controllers have the following drawbacks[1].

1) Application limitations, like only reliable for delay smaller than process time constant.
2) Sensitive to noise.
3) Not suitable for nonlinear interactive models.

We analyze PID controller under several delays by measuring system performance parameters (stability margins, %overshoots, settling times, and error signals) with a plant modeled by order one with time delay using two tuning methods, iterative method[1] and Ziegler-Nichols method[4]. Although we use rough plant estimation, actually it can be applied in many cases because this sort of equation can model most of industrial processes. And for the sake of simulation, we use $G(s) = 1/(s+1)$ and $\tau_p = 0.5$.

By comparing Table 3 with Table 2, iterative method brings significant improvements in all system performance parameters in average. Stability margin improves more than three times, %overshoot reduces half, and settling time reduces more than 20%.

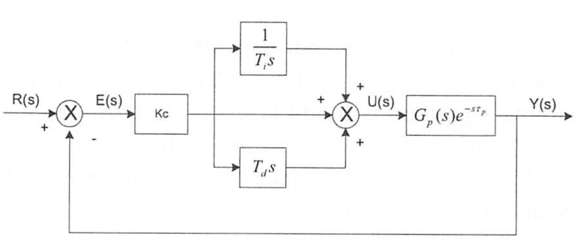

Figure 5. PID controller general structure

Table 2. System performance parameters without PID controller

Delay	Stability Margin (SM)	% Overshoot (%O)	Settling Time (ST)	Error Signal (ES)
0.01	7.2	0	1.45	0.5
0.025	0.7	0	1.44	0.5
0.05	7.8	0	1.44	0.5
0.075	9.3	0	1.43	0.5
0.1	11.5	0	1.41	0.5
0.25	7.8	0	1.29	0.5
0.5	4	8.62	2.06	0.5
0.75	2.8	24.3	3.66	0.5
1	2.2	38	4.88	0.5
Av.	5.92	7.88	2.12	0.5

Table 3. System performance parameters with iterative method (three times)

Delay	K	Ti	Td	SM	% Ov	ST	ES
0.01	3.9	0.9	0.01	64.7	0.7	0.59	-
0.025	3.29	1	0.02	30.3	0	0.82	-
0.05	3.83	1	0.02	25	0	0.66	-
0.075	4.56	1	0.018	16.5	0.5	0.33	-
0.1	5.65	1.5	0.02	13.7	6.2	0.77	-
0.25	3.95	0.876	0.108	8.49	9.9	0.68	-
0.5	2	1.35	0.2	4.3	10	2.4	-
0.75	1.43	1.74	0.243	3	4.5	3.45	-
1	1.05	1.98	0.283	2.4	2.56	4.71	-
Av.				18.71	3.82	1.60	-

Table 4. System performance parameters with Ziegler-Nichols method

Delay	Kcr	Pcr	Kp	Ti	Td	SM	% Ov	ST	G
0.01	6.08	0.4	3.65	0.2	0.05	14.4	28.46	1.12	6.33
0.025	6.59	0.4	3.95	0.2	0.05	13.7	31.08	1.03	1.78
0.05	7.66	0.4	4.6	0.2	0.05	9	42.94	1.18	1.04
0.075	9.12	0.4	5.47	0.2	0.05	5.3	39.5	-	1.03
0.1	11.3	0.4	6.78	0.2	0.05	6.32	44.72	-	1.05
0.25	7.9	1	4.74	0.5	0.125	7.43	36.61	1.34	3.68
0.5	4	1.7	2.4	0.85	0.213	4.33	37.47	2.57	2.13
0.75	2.84	2.4	1.72	1.2	0.3	2.87	34.07	3.68	2
1	2.3	3.08	1.38	1.5	0.385	2.25	31.48	4.71	2
Av.						7.29	36.26	2.23	

In Table 4, settling times at delay 0.075 and 0.10 second can't be measured, because system responses are oscillating. If we compare these results with iterative method, we can see that stability margin is smaller and %overshoot bigger. This implies that Ziegler-Nichols method producmakes systems less stable and take more time to settle.

5. Conclusions

This study is summarized as follows :

1. In the analyzed cases, we show that Direct Frequency Response series has the best overall performance among the others series.
2. There is a breaking point in such if time delay is smaller than this value all seven series give relatively the same values, and if bigger than this value they will diverge especially in cases of order two and three.
3. It is advisable not to use Marshall and Taylor series in delay approximations because these two series have bigger error rates comparing to others five series in all case.
4. Two tuning methods, Iterative Method and Ziegler-Nichols Method, are analyzed for PID controllers, and it confidently shows that Iterative Method has superior performance.

References

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